

NAG C Library Function Document

nag_poisson_ci (g07abc)

1 Purpose

nag_poisson_ci (g07abc) computes a confidence interval for the mean parameter of the Poisson distribution.

2 Specification

```
void nag_poisson_ci (Integer n, double xmean, double clevel, double *tl, double *tu,
                    NagError *fail)
```

3 Description

Given a random sample of size n , denoted by x_1, x_2, \dots, x_n , from a Poisson distribution with probability function

$$p(x) = e^{-\theta} \frac{\theta^x}{x!}, \quad x = 0, 1, 2, \dots$$

the point estimate, $\hat{\theta}$, for θ is the sample mean, \bar{x} .

Given n and \bar{x} this function computes a $100(1 - \alpha)\%$ confidence interval for the parameter θ , denoted by $[\theta_l, \theta_u]$, where α is in the interval $(0,1)$.

The lower and upper confidence limits are estimated by the solutions to the equations

$$e^{-n\theta_l} \sum_{x=T}^{\infty} \frac{(n\theta_l)^x}{x!} = \frac{\alpha}{2},$$

$$e^{-n\theta_u} \sum_{x=0}^T \frac{(n\theta_u)^x}{x!} = \frac{\alpha}{2},$$

where $T = \sum_{i=1}^n x_i = n\hat{\theta}$.

The relationship between the Poisson distribution and the χ^2 distribution (see page 112 of Hastings and Peacock (1975)) is used to derive the equations

$$\theta_l = \frac{1}{2n} \chi_{2T, \alpha/2}^2,$$

$$\theta_u = \frac{1}{2n} \chi_{2T+2, 1-\alpha/2}^2,$$

where $\chi_{\nu, p}^2$ is the deviate associated with the lower tail probability p of the χ^2 distribution with ν degrees of freedom.

In turn the relationship between the χ^2 distribution and the gamma distribution (see page 70 of Hastings and Peacock (1975)) yields the following equivalent equations;

$$\theta_l = \frac{1}{2n} \gamma_{T, 2; \alpha/2},$$

$$\theta_u = \frac{1}{2n} \gamma_{T+1, 2; 1-\alpha/2},$$

where $\gamma_{\alpha, \beta; \delta}$ is the deviate associated with the lower tail probability, δ , of the gamma distribution with shape parameter α and scale parameter β . These deviates are computed using nag_deviates_gamma_dist (g01ffc).

4 References

Hastings N A J and Peacock J B (1975) *Statistical Distributions* Butterworths

Snedecor G W and Cochran W G (1967) *Statistical Methods* Iowa State University Press

5 Parameters

- | | | |
|----|--|---------------------|
| 1: | n – Integer | <i>Input</i> |
| | <i>On entry:</i> the sample size, n . | |
| | <i>Constraint:</i> $n \geq 1$. | |
| 2: | xmean – double | <i>Input</i> |
| | <i>On entry:</i> the sample mean, \bar{x} . | |
| | <i>Constraint:</i> xmean ≥ 0.0 . | |
| 3: | clevel – double | <i>Input</i> |
| | <i>On entry:</i> the confidence level, $(1 - \alpha)$, for two-sided interval estimate. For example clevel = 0.95 gives a 95% confidence interval. | |
| | <i>Constraint:</i> $0.0 < \mathbf{clevel} < 1.0$. | |
| 4: | tl – double * | <i>Output</i> |
| | <i>On exit:</i> the lower limit, θ_l , of the confidence interval. | |
| 5: | tu – double * | <i>Output</i> |
| | <i>On exit:</i> the upper limit, θ_u , of the confidence interval. | |
| 6: | fail – NagError * | <i>Input/Output</i> |
| | The NAG error parameter (see the Essential Introduction). | |

6 Error Indicators and Warnings

NE_INT

On entry, **n** = $\langle value \rangle$.

Constraint: **n** > 0 .

NE_CONVERGENCE

When using the relationship with the gamma distribution the series to calculate the gamma probabilities has failed to converge.

NE_REAL

On entry, **clevel** ≤ 0.0 or **clevel** ≥ 1.0 : **clevel** = $\langle value \rangle$.

On entry, **xmean** = $\langle value \rangle$.

Constraint: **xmean** ≥ 0.0 .

NE_BAD_PARAM

On entry, parameter $\langle value \rangle$ had an illegal value.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

7 Accuracy

For most cases the results should have a relative accuracy of $\max(0.5e-12, 50.0 \times \epsilon)$ where ϵ is the *machine precision*. Thus on machines with sufficiently high precision the results should be accurate to 12 significant digits. Some accuracy may be lost when $\alpha/2$ or $1 - \alpha/2$ is very close to 0.0, which will occur if **clevel** is very close to 1.0. This should not affect the usual confidence intervals used.

8 Further Comments

None.

9 Example

The following example reads in data showing the number of noxious weed seeds and the frequency with which that number occurred in 98 sub-samples of meadow grass. The data is taken from page 224 of Snedecor and Cochran (1967). The sample mean is computed as the point estimate of the Poisson parameter θ . `nag_poisson_ci` (g07abc) is then called to compute both a 95% and a 99% confidence interval for the parameter θ .

9.1 Program Text

```

/* nag_poisson_ci (g07abc) Example Program.
 *
 * Copyright 2001 Numerical Algorithms Group.
 *
 * Mark 7, 2001.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagg07.h>

int main(void)
{
    /* Scalars */
    double clevel, sum, tl, tu, xmean;
    Integer exit_status, i, ifreq, n, num;
    NagError fail;

    INIT_FAIL(fail);
    exit_status = 0;
    Vprintf("g07abc Example Program Results\n");

    /* Skip heading in data file */
    Vscanf("%*[\n] ");

    /* Read in the number of Noxious Seeds in a sub sample and
     * the frequency with which that number occurs.
     */

    /* Compute the sample mean */
    sum = 0.0;
    n = 0;

    while (scanf("%ld%ld%*[\n] ", &num, &ifreq) != EOF)
    {
        sum += (double) num * (double) ifreq;
        n += ifreq;
    }
    xmean = sum / (double) n;

    Vprintf("\n");
    Vprintf("The point estimate of the Poisson parameter = %6.4f\n", xmean);
    for (i = 1; i <= 2; ++i)

```

```
{
  if (i == 1)
  {
    clevel = 0.95;
    Vprintf("\n");
    Vprintf("95 percent Confidence Interval for the estimate\n");
  }
  else
  {
    clevel = 0.99;
    Vprintf("99 percent Confidence Interval for the estimate\n");
  }
  g07abc(n, xmean, clevel, &t1, &tu, &fail);
  if (fail.code != NE_NOERROR)
  {
    Vprintf("Error from g07abc.\n%s\n", fail.message);
    exit_status = 1;
    goto END;
  }

  Vprintf("( %6.4f , %6.4f )\n", t1, tu);
  Vprintf("\n");
}

END:
return exit_status;
}
```

9.2 Program Data

```
g07abc Example Program Data
0 3
1 17
2 26
3 16
4 18
5 9
6 3
7 5
8 0
9 1
10 0
```

9.3 Program Results

```
g07abc Example Program Results
```

```
The point estimate of the Poisson parameter = 3.0204
```

```
95 percent Confidence Interval for the estimate
( 2.6861 , 3.3848 )
```

```
99 percent Confidence Interval for the estimate
( 2.5874 , 3.5027 )
```
